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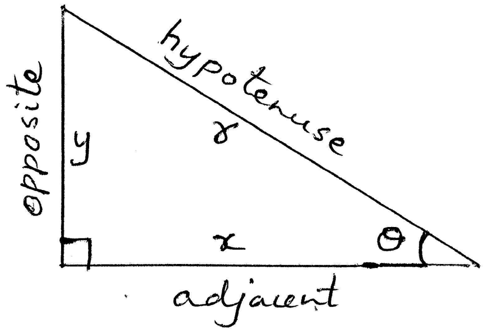
Trignometric functions



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{r}{y}, \quad \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}, \quad \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

If x is an angle in degrees and θ an angle in radians then

$$\frac{\pi}{180} = \frac{\theta}{x} \implies \theta = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180\theta}{\pi}$$

$$y = \sin^{-1}x \Rightarrow x = \sin y$$

$$y = \cos^{-1}x \Rightarrow x = \cos y$$

$$y = \tan^{-1}x \Rightarrow x = \tan y$$

$$\cos[\cos^{-1}(\theta)] = \theta \qquad \cos^{-1}[\cos(\theta)] = \theta$$

$$\sin[\sin^{-1}(\theta)] = \theta \qquad \sin^{-1}\{\sin(\theta)] = \theta$$

$$\tan(\tan^{-1}(\theta)] = \theta \qquad \tan^{-1}[\tan(\theta)] = \theta$$

$$\sin^{-1}x + \cos^{-1}x = \pi/2$$

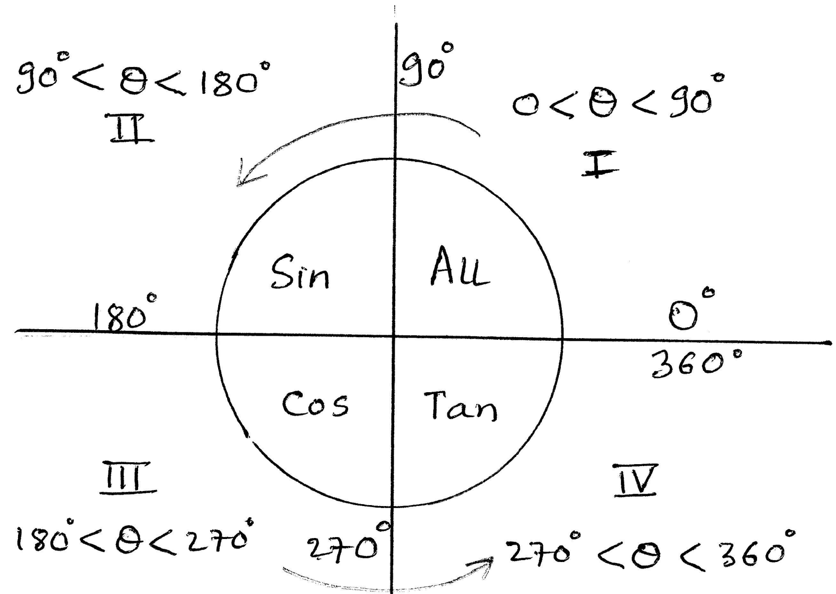
$$\tan^{-1}x + \cot^{-1}x = \pi/2$$

$$\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$$

	0°	30°	45°	60°	90°
sine	$\sqrt{0}/2 = 0$	$\sqrt{1}/2 = \frac{1}{2}$	$\sqrt{2}/2 = \frac{1}{\sqrt{2}}$	$\sqrt{3}/2 = \frac{\sqrt{3}}{2}$	$\sqrt{4}/2 = 1$
cos	$\sqrt{4}/2 = 1$	$\sqrt{3}/2 = \frac{\sqrt{3}}{2}$	$\sqrt{2}/2 = \frac{1}{\sqrt{2}}$	$\sqrt{1}/2 = \frac{1}{2}$	$\sqrt{0}/2 = 0$
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

ASTC Rule

All Students Take Coffee



- Trignometric ratios of $90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$, $360 \pm \theta$ can be found easily by applying ASTC Rule.
- When the angle is $90 \pm \theta$ or $270 \pm \theta$, the trignometric ratio changes from sine to cosine, tan to cot, sec to cosec and vice versa.
- When the angle is $180 \pm \theta$ or $360 \pm \theta$, the trignometric ratio remains the same ie sine $\rightarrow$ sine, cos $\rightarrow$ cos etc..
- In each case the sign + or − is premultiplied by ASTC quadrant Rule.

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Differentiation

$$\frac{d}{dx}(\text{const}) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = a^x \log a$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 hx$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x . \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\operatorname{cosech}^{-1} x) = \frac{-1}{x\sqrt{1+x^2}}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$ (Product Rule)

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$ (Quotient Rule)

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Trignometry Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A . \tan B}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\sin 2\theta = 2\sin\theta . \cos\theta = 2\tan\theta / 1 + \tan^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1 = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin A + \sin B = 2\sin\left[\frac{A+B}{2}\right].\cos\left[\frac{A-B}{2}\right]$$

$$\sin A - \sin B = 2\cos\left[\frac{A+B}{2}\right].\sin\left[\frac{A-B}{2}\right]$$

$$\cos A + \cos B = 2\cos\left[\frac{A+B}{2}\right].\cos\left[\frac{A-B}{2}\right]$$

$$\cos A - \cos B = -2\sin\left[\frac{A+B}{2}\right].\sin\left[\frac{A-B}{2}\right]$$

$$\sin A.\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A.\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A.\cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\operatorname{sech}^2 x + \tanh^2 x = 1$$

$$\coth^2 x - \operatorname{cosech}^2 x = 1$$

$$\sin^2 \theta/2 + \cos^2 \theta/2 = 1$$

$$\sin\theta = 2\sin\theta/2 . \cos\theta/2$$

$$\cos 2\theta = \cos^2\theta/2 - \sin^2\theta/2$$

$$1 - \cos\theta = 2\sin^2\theta/2$$

$$1 + \cos\theta = 2\cos^2\theta/2$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1+\tan\theta}{1-\tan\theta} = \tan\left(\pi/4 + \theta\right)$$

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Logarithms

$e^0 = 1$

$e = 2.718281845 9045 \ldots \simeq 2.73$

inverse of e^x is $\ln(x)$ or natural logarithm

$\ln(xy) = \ln x + \ln y$

$\ln(x/y) = \ln x - \ln y$

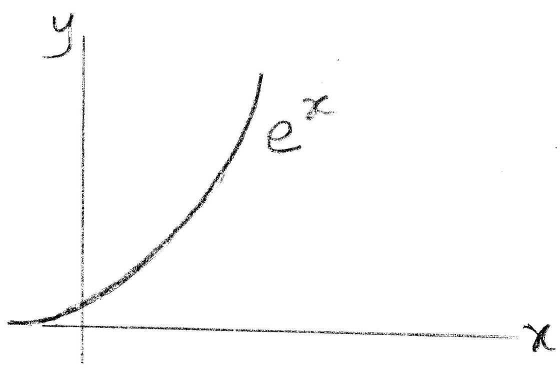
$\ln x^y = y \ln x$

$\ln(e^x) = x$

$e^{\ln x} = x$

$e^{-\ln x} = e^{\ln(1/x)} = 1/x$

Base 10 logarithms are most commonly used.
written as $\log_{10} x$ or just $\log x$



$10^{\log x} = x$

$10^{-\log x} = 1/x$

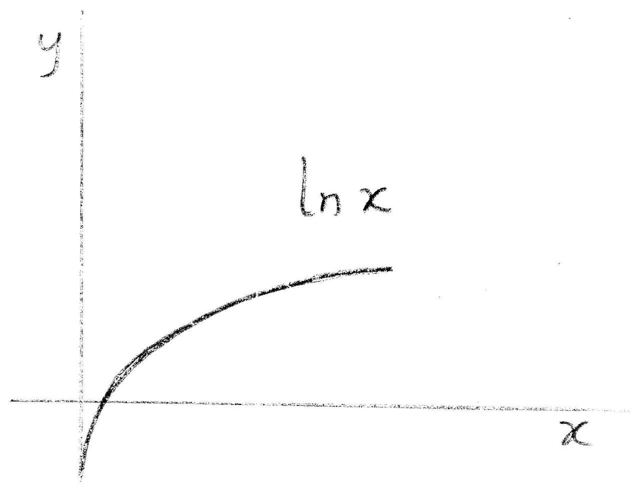
$\log(xy) = \log x + \log y$

$\log(x/y) = \log x - \log y$

$\log x^y = y \log x$

$\log(10^x) = x$

$\log_b a = \dfrac{\log_e a}{\log_e b} = \dfrac{\log_{10} a}{\log_{10} b}$



$e^{\log x} = x \qquad \log_a a = 1$

$e^{-\log x} = 1/x \qquad \log_a 1 = 0$

$\log(e^x) = x \qquad \log_e 0 = -\infty$

$a^x = e^{(\log a)x}$

Partial fractions

The degree of the Numerator must be less than the degree of Denominator.

- for a linear term $(ax+b)$ we get $\frac{A}{ax+b}$
- for a repeated linear term such as $(ax+b)^3$ we get $\frac{A}{(ax+b)} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3}$
- for the quadratic term ax^2+bx+c we get $\frac{Ax+B}{ax^2+bx+c}$

Eg:

$$\frac{2x^2+1}{(x+2)(x+3)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x+3)} + \frac{C}{(x-1)}$$

$$\frac{x^2-1}{(x+7)(x+2)^2} = \frac{A}{(x+7)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$\frac{7x}{(x^2+x+2)(x+2)} = \frac{Ax+B}{x^2+x+2} + \frac{C}{(x+2)}$$

For improper fraction we convert it to a sum of polynomial and a proper fraction by long division.

E.g. $\frac{2x^3+3x^2-x+1}{x^2-x-2} \Longrightarrow$

$$\begin{array}{r|l} & 2x+5 \\ \hline x^2-x-2 & 2x^3+3x^2-x+1 \\ & 2x^3-2x^2-4x \\ \hline & 5x^2+3x+1 \\ & 5x^2-5x-10 \\ \hline & 8x+11 \end{array}$$

$$\therefore \frac{2x^3+3x^2-x+1}{x^2-x-2} = (2x+5) + \frac{8x+11}{x^2-x-2}$$

Quotient + Remainder (a proper fraction)

$\Downarrow$

Resolved using above.

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Integration

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int \frac{1}{x} \, dx = \log x, \quad \int \frac{1}{x^2} \, dx = \frac{-1}{x}$$

$$\int e^x \, dx = e^x$$

$$\int a^x \, dx = \frac{a^x}{\log a}$$

$$\int \sin x \, dx = -\cos x$$

$$\int \cos x \, dx = \sin x$$

$$\int \tan x \, dx = \log(\sec x)$$

$$\int \sec x \, dx = \log(\sec x + \tan x)$$

$$\int \cot x \, dx = \log(\sin x)$$

$$\int \operatorname{cosec} x \, dx = \log(\operatorname{cosec} x - \cot x)$$

$$\int \sec^2 x \, dx = \tan x$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x$$

$$\int \sec x . \tan x \, dx = \sec x$$

$$\int \sinh x \, dx = \cosh x$$

$$\int \cosh x \, dx = \sinh x$$

$$\int \tanh x \, dx = \log \cosh x$$

$$\int \coth x \, dx = \log \sinh x$$

$$\int \operatorname{sech} x \cdot \tanh x \, dx = -\operatorname{sech} x$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left[\frac{x-a}{x+a}\right]$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left[\frac{a+x}{a-x}\right]$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left[\frac{x}{a}\right]$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) = \log\left(\frac{x+\sqrt{x^2+a^2}}{a}\right)$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right) = \log\left[\frac{x+\sqrt{x^2-a^2}}{a}\right]$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left[\frac{x}{a}\right]$$

$$\int e^{ax}\sin(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin(bx+c) - b\cos(bx+c)\right)$$

$$\int e^{ax}\cos(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos(bx+c) + b\sin(bx+c)\right)$$

$\int 1.2 = 1\int 2 - \int\int 2.\frac{d}{dx}(1)$ – integration by parts

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$ Bernoulis Theorem

$u', u'', u''' \cdots$ derivatives, $v_1, v_2, v_3 \cdots$ integrals

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n^{th} Derivatives

Function	n^{th} derivative
$y = e^{ax}$	$y_n = a^n e^{ax}$
$y = a^{mx}$	$y_n = (m \log a)^n a^{mx}$
$y = (ax+b)^m$	• if m is integer greater than n or less than -1, then $y_n = m(m-1)(m-2)\ldots\ldots(m-n+1)a^n(ax+b)^{m-n}$ • if m is less than n, $y_n = 0$ • if $m = n$, then $y_n = a^n . n!$ • if $m = -1$ then $y_n = \dfrac{(-1)^n n! a^n}{(ax+b)^{n+1}}$ • if $m = -2$ then $y_n = \dfrac{(-1)^n (n+1)! a^n}{(ax+b)^{n+1}}$
$y = \log(ax+b)$	$y_n = \dfrac{(-1)^{n-1}(n-1)! a^n}{(ax+b)^n}$
$y = \dfrac{1}{(ax+b)}$	$y_n = \dfrac{(-1)^n n! a^n}{(ax+b)^{n+1}}$
$y = \sin(ax+b)$	$y_n = a^n \sin\left[\frac{n\pi}{2} + ax + b\right]$
$y = \cos(ax+b)$	$y_n = a^n \cos\left[\frac{n\pi}{2} + ax + b\right]$
$y = e^{ax}\sin(bx+c)$	$(\sqrt{a^2+b^2})^n e^{ax} \sin[n \tan^{-1}(b/a) + bx + c]$
$y = e^{ax}\cos(bx+c)$	$(\sqrt{a^2+b^2})^n e^{ax} \cos[n \tan^{-1}(b/a) + bx + c]$

Leinitz Theorem (n^{th} derivative of a product)

$$(uv)_n = uv_n + nu_1v_{n-1} + \frac{n(n-1)}{2!}u_2v_{n-2} + \frac{n(n-1)(n-2)}{3!}u_3v_{n-3} + \frac{n(n-1)(n-2)(n-3)}{4!}u_4v_{n-4} + \cdots\cdots + u_nv$$

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Differential Equations (Higher order)

D → differential operator

$D \to dx \qquad \frac{1}{D} \to \int dx$

Order → highest derivative

degree → power of highest derivative

E.g; $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = \sin x$ order is 2, degree is 3

Homogeneous → the function is zero : $3\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0$

Non homogenous → the function is not zero : $4\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^x$

Solution of D.E y = complementary factor + particular integral

(C.F) + (P.I)

(depends on roots) (depends on function)

Finding C.F

$FD(y) = \phi(x)$ ← function

auxillary eqn is $f(m) = 0$

find the roots

$m = m_1, m_2, m_3, \ldots$

E.g $4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = \cos x$

$(4D^2 + D + 6)y = \cos x$

A.E is $4m^2 + m + 6 = 0$

- roots real and different; say $m = 2, 3, -5$

  $C.F = c_1 e^{2x} + c_2 e^{3x} + c_3 e^{-5x}$
- roots are coincident or same; say $m = 2, 2$

  $C.F = (c_1 + c_2 x)e^{2x}$ or say $m = -3, -3, -3$

  $C.F = (c_1 + c_2 x + c_3 x)e^{-3x}$
- roots are complex conjugate say $3 \pm 2i$

  $C.F = (c_1 \cos 2x + c_2 \sin 2x)e^{3x}$
- roots are pure imaginary say $m = \pm 2i$

  $C.F = (c_1 \cos 2x + c_2 \sin 2x)$

Finding the Particular Integral

$$y_p = \frac{e^{ax}}{f(D)}$$

Replace $D \to a$

$$\frac{e^{ax}}{f(a)}$$

if $f(a) \neq 0$ then

$$y_p = \frac{e^{ax}}{f(a)}$$

$\downarrow$ P.I

if $f(a) = 0$ then

$$\frac{x e^{ax}}{f'(D)}$$

$D \to a$

if $f'(a) = 0$ then

$$\frac{x^2 e^{ax}}{f''(D)}$$

$D \to a$

so on

Note: x is premultiplied i.e; it is kept as it is and is not differentiated or integrated

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$$y_p = \frac{\sin ax \text{ or } \cos ax}{f(D)}$$

Replace $D^2 \to -a^2$

$$\frac{\sin ax}{f(-a^2)}$$

if $f(-a^2) \neq 0$ then

$$y_p = \frac{\sin ax}{f(-a^2)}$$

$\downarrow$ P.I

if $f(-a^2) = 0$ then

$$\frac{x \sin ax}{f'(D)}$$

$D^2 \to -a^2$

if $f'(-a^2) = 0$ then

$$\frac{x^2 \sin ax}{f''(D)}$$

$D^2 \to -a^2$

* Dr needs to have a D^2 term if not then rationalize

E.g $\frac{\sin 2x}{D+2} \Rightarrow \frac{\sin 2x\,(D-2)}{(D+2)(D-2)}$

$\Rightarrow \frac{\sin 2x\,(D-2)}{D^2-4}$ $\quad D^2 \to -4$

* x is premultiplied

$$y_p = \frac{\phi(x)}{f(D)} \to \text{polynomial}$$

$$y_p = \frac{\phi(x)}{f(D)} \begin{matrix} \to \text{descending} \\ \to \text{ascending} \end{matrix}$$

P.I = Quotient + remainder ($\to 0$)

$\Rightarrow$ P.I = Quotient

$$y_p = \frac{e^{ax} V(x)}{f(D)} \qquad V(x) \to \text{any function}$$

Replace $D \to D+a$

$$= e^{ax}\left(\frac{V(x)}{f(D+a)}\right)$$

Solve $\left[\frac{V(x)}{f(D+a)}\right]$ by the methods described above.

Note: e^{ax} is premultiplied

$$\text{P.I} = e^{ax}\left[\text{soln of } \frac{V(x)}{f(D+a)}\right]$$

$$y_p = \frac{x\,V(x)}{f(D)}$$

$$\text{P.I} = \left[x - \frac{f'(D)}{f(D)}\right]\left[\frac{V(x)}{f(D)}\right]$$

first solve for $\left[\frac{V(x)}{f(D)}\right]$ then remove the brackets and Solve the new D.E.

In case of $\frac{x^n \sin ax}{f(D)}$ or $\frac{x^n \cos ax}{f(D)}$

let $e^{iax} = \cos ax + i \sin ax$

($\cos ax$: Real part; $\sin ax$: imaginary part)

Then,

$$y_p = \text{R.P}\,\frac{e^{iax} \cdot x^n}{f(D)} \text{ or } \text{I.P}\,\frac{e^{iax} x^n}{f(D)}$$

$$= \text{R.P}\,e^{iax}\left(\frac{x^n}{f(D)}\right)_{D \to D+ai} \quad ''$$

Solve

After solving replace $e^{iax} \to \cos ax + i \sin ax$ and remove brackets then,

P.I = All real terms (for $\frac{x^n \cos ax}{f(D)}$)

= All imaginary terms with i (for $\frac{x^n \sin ax}{f(D)}$)

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Inverse Laplace Transform

We know how to find laplace transforms of unknown functions satisfying various initial value problems. It's not the transforms of those unknown functions which are usually of interest. It's the functions, themselves, that are of interest. So here, we find a function $f(t)$ when all we know is it's Laplace transform $\bar{f}(s)$.

$L^{-1}[\bar{f}(s)] = f(t)$ is the inverse laplace transform.

$\bar{f}(s)$	$L^{-1}[\bar{f}(s)]$	$\bar{f}(s)$	$L^{-1}[\bar{f}(s)]$
$\frac{1}{s}$	1	$\frac{1}{s-a}$	e^{at}
$\frac{1}{s^2-a^2}$	$\frac{\sinh at}{a}$	$\frac{s}{s^2-a^2}$	$\frac{\cosh at}{a}$
$\frac{1}{s^2+a^2}$	$\frac{1}{a}\sin at$	$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{1}{s^{n+1}}$ $(n>-1)$	$\frac{t^n}{\Gamma(n+1)}$	$\frac{1}{s^{n+1}}$ $n=1,2,3$	$\frac{t^n}{n!}$

properties

- $L^{-1}[e^{-as}\bar{f}(s)] = f(t-a)\,u(t-a)$
- $L^{-1}[\bar{f}(s-a)] = e^{at}L^{-1}[\bar{f}(s)]$
- $L^{-1}[-\bar{f}'(s)] = t\,f(t)$

  $L^{-1}[\bar{f}''(s)] = t^2 f(t)$
- $L^{-1}\left[\frac{\bar{f}(s)}{s}\right] = \int_0^t f(t)\,dt$

Convolution

Convolution is a mathematical way of combining two signals to form a third signal. It is important because it relates the three signals of interest, the i/p signal, the o/p signal, and the impulse response. denoted by operator $*$.

For two functions $f(t)$ and $g(t)$

$$f(t) * g(t) = \int_{u=0}^{t} f(u)\, g(t-u)\, du$$

$$f(t) * g(t) = g(t) * f(t)$$

Convolution Theorem.

If $L^{-1}[\bar{f}(s)] = f(t)$ and $L^{-1}[\bar{g}(s)] = g(t)$, then

$$L^{-1}[\bar{f}(s) \cdot \bar{g}(s)] = \int_{u=0}^{t} f(u)\, g(t-u)\, du = f(t) * g(t)$$

or

$$L\left[\int_0^t f(u)\, g(t-u)\, du\right] = \bar{f}(s) \cdot \bar{g}(s) = L\left[\int_0^t f(t-u)\, g(u)\, du\right]$$

Laplace Transforms of derivatives

- $L[y'(t)] = s\,Ly(t) - y(0)$
- $L[y''(t)] = s^2 Ly(t) - sy(0) - y'(0)$
- $L[y'''(t)] = s^3 Ly(t) - s^2 y(0) - sy'(0) - y''(0)$
- $L[y^n(t)] = s^n L[y(t)] - \sum_{k=1}^{n} s^{k-1} y^{n-k}(0)$

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Laplace Transform

Laplace Transform is a widely used integral transform in mathematics, physics and engineering applications. By using this we can solve an equation or a system of equations containing differential and integral terms by transforming the equation in "t-space" (time domain where all inputs and outputs are functions of time $f(t)$) to the "s-space" (where the same i/p and o/p's are functions of complex frequency s, $\bar{f}(s)$).

$$L[f(t)] = \int_0^{\infty} e^{-st} f(t)\, dt = \bar{f}(s)$$

$f(t)$	$L[f(t)]$	$f(t)$	$Lf(t)$
a	$\frac{a}{s}$	e^{at}	$\frac{1}{s-a}$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\cosh at$	$\frac{s}{s^2-a^2}$
$\sin at$	$\frac{a}{s^2+a^2}$	$\cos at$	$\frac{s}{s^2+a^2}$
t^n	$\frac{\Gamma(n+1)}{s^{n+1}}$	t^n $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$

Properties

- $L[e^{at} f(t)] = L[f(t)]_{s \to s-a} = \bar{f}(s-a)$ → first shift formula
- $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[Lf(t)] = (-1)^n \frac{d^n}{ds^n}[\bar{f}(s)]$ → second differentiation formula
- $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} L[f(t)] = \int_s^{\infty} \bar{f}(s)\, ds$
- $L\left[\int_0^t f(t)\, dt\right] = \frac{L[f(t)]}{s} = \frac{\bar{f}(s)}{s}$

Periodic function

If a function $f(t)$ is periodic with a period $T>0$ so that $f(t+T)=f(t)$

$$L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$$

Unit step function (Heaviside function)

The unit step function is like an off-ON switch that turns a function ON or OFF over a specified interval, By definition.

$$u(t-a) \text{ or } H(t-a) = \begin{cases} 0, & 0<t\leq a \\ 1, & t>a \end{cases}$$

- $L[u(t-a)] = \dfrac{e^{-as}}{s}$
- $L[f(t-a)\,u(t-a)] = e^{-as}\bar{f}(s) = e^{-as}L[f(t)]$
- if $f(t) = \begin{cases} f_1(t), & t\leq a \\ f_2(t), & t>a \end{cases}$

  Then $f(t) = f_1(t) + [f_2(t)-f_1(t)]\,u(t-a)$
- if $f(t) = \begin{cases} f_1(t), & t\leq a \\ f_2(t), & a<t\leq b \\ f_3(t), & t>b \end{cases}$

  Then $f(t) = f_1(t) + [f_2(t)-f_1(t)]\,u(t-a) + [f_3(t)-f_2(t)]\,u(t-b)$

Unit impulse function (Dirac delta function)

The delta function is used to model "instantaneous" energy transfers. (applying a Large force over a Small time frame ; e.g; insted of turning on a voltage, just short it)

$$\delta(t-a) = \lim_{\varepsilon\to 0}\delta_\varepsilon(t-a) \;;\; a>0$$

where $\delta_\varepsilon(t-a) = \begin{cases} \frac{1}{\varepsilon} & \text{if } a\leq t\leq a+\varepsilon \\ 0 & \text{otherwise} \end{cases}$

$$L[\delta(t-a)] = e^{-as}$$